JHEP HYPER VERSION

Received: *December 19, 2000*, Accepted: *March 19, 2001*
Revised: *March 14, 2001*

The Kałuża-Klein Melvin solution in M-theory

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Abstract: We study some aspects of the Kałuża-Klein Melvin solution in M-theory. The associated magnetic field has a maximal critical value $B = \pm 1/R$ where R is the radius of the compactification circle. It is argued that the Melvin background of type IIA with magnetic field B and of type 0A with magnetic field $B' = B - 1/R$ are equivalent. Evidence for this conjecture is provided using a further circle compactification and a '9-11' flip. We show that partition functions of nine-dimensional type-IIA strings and of a $(-1)^F \sigma_{1/2}$ type-IIA orbifold both with NS-NS Melvin flux-tubes are related by such shift of the magnetic field. Then the instabilities of both IIA and 0A Melvin solutions are analyzed. For each theory there is an instanton associated to the decay of spacetime. In the IIA case the decay mode is associated to the nucleation of D6/D$\bar{6}$-brane pairs, while in the 0A case spacetime decays through Witten's bubble production.

Keywords: Superstrings and Heterotic Strings, M-Theory, Superstring Vacua.

JHEP03(2001)027

Contents

1. Introduction

The study of string theories in strong background fields has received considerable attention over the past years. For example, in the context of the AdS/CFT duality, the near horizon limit of D3-branes gives IIB string theory in an $AdS_5 \times S_5$ background with self-dual R-R 5-form flux [1]. More recently, open strings and membranes in near critical electric fields were seen to produce new interesting dynamical regimes in string theory and M-theory [2]–[5].

In this paper some aspects of another example of a string background with nontrivial flux will be discussed. The Melvin fluxtube universe [6] describes an axisymmetric spacetime with a non-vanishing magnetic flux along the axis. Interestingly such a spacetime can be realized as the Kałuża-Klein reduction from a flat higher dimensional spacetime with nontrivial identifications. In a nice series of papers [7]–[10], Dowker et al. analyzed the Kałuża-Klein Melvin solution. In particular, they discussed the nonperturbative instability and pair production of magnetically charged black holes. In this work we apply some of their results in the M-theory context.

The plan of this paper is as follows. In section two we describe the Kałuża-Klein Melvin solution of IIA strings coming from dimensional reduction of M-theory.

In particular, we discuss the appearance of a maximal magnetic field and the issue of spin structures. The nonzero magnetic flux in the Kałuża-Klein Melvin solution breaks all the supersymmetries. We shall argue that this Melvin solution is equivalent to 0A string theory [11, 12] in a (different) magnetic field. A '9-11' flip relates these two theories to the nine-dimensional type-II strings and to the $S_1/(-1)^F\sigma_{1/2}$ type-IIA orbifold both with a NS-NS Melvin fluxtube. In this case the equivalence of both theories can be analyzed explicitly.

In section three the instabilities of the Melvin background are discussed. These are associated to instantons describing decay modes of the spacetime. There are two instantons, namely the shifted and unshifted instantons, which differ by their spin structures and give the decay modes for the Melvin IIA and 0A string backgrounds, respectively. In the former case the instanton is associated to the nucleation of D6/D$\bar{6}$-brane pairs while in the latter to a deformed version of Witten's expanding bubble [13].

We give our conclusions in section four.

2. Melvin Solution from M-theory

An interesting solution of Einstein-Maxwell theory is the Melvin solution [6] which represents a magnetic fluxtube universe. One of the most surprising features of the Kałuża-Klein Melvin solution [14] is that the corresponding higher dimensional spacetime is flat but has nontrivial identifications [8, 9]. Consider the eleven-dimensional flat metric in M-theory written in cylindrical coordinates

$$ds^2 = -dt^2 + dy_m dy^m + dz^2 + d\rho^2 + \rho^2 d\varphi^2 + {dx_{11}}^2 \,, \tag{2.1}$$

where $m = 1, \dots, 6$. For simplicity the six-dimensional manifold spanned by y^m is taken to be a six-torus. Next make the following identification:

$$(t, y_m, z, \rho, \varphi, x_{11}) \equiv (t, y_m, z, \rho, \varphi + 2\pi n_1 RB + 2\pi n_2, x_{11} + 2\pi n_1 R) \,, \tag{2.2}$$

for every integer n_1 and n_2. The identifications under the shifts $2\pi n_1 R$ for x_{11} and $2\pi n_2$ for φ are standard. However, the shift $x_{11} \to x_{11} + 2\pi n_1 R$ is accompanied by a rotation $\varphi \to \varphi + 2\pi n_1 RB$. Upon dimensional reduction along the Killing vector

$$l = \partial_{11} + B\,\partial_\varphi \tag{2.3}$$

the above identifications give rise to the dilatonic Melvin background [14]. It is convenient to introduce the coordinate $\tilde{\varphi} = \varphi - B\,x_{11}$, which is constant along orbits of l and has standard periodicity. Using the relation between the M-theory metric and the string frame metric, dilaton field and R-R 1-form potential[1]

$${ds_{11}}^2 = e^{-2\phi/3} {ds_{10}}^2 + e^{4\phi/3} \left(dx_{11} + 2\mathcal{A}_\mu dx^\mu\right)^2 \,, \tag{2.4}$$

[1]Notice for future reference that with the factor of 2 multiplying the R-R 1-form potential, the D6-brane tension T_6 and charge density ρ_6 are related by $\rho_6 = 2T_6$.

the ten-dimensional IIA background is described by

$$ds_{10}{}^2 = \Lambda^{1/2}\left(-dt^2 + dy_m dy^m + dz^2 + d\rho^2\right) + \Lambda^{-1/2}\rho^2 d\tilde{\varphi}^2\,,$$
$$e^{4\phi/3} = \Lambda \equiv 1 + \rho^2 B^2\,, \qquad \mathcal{A}_{\tilde{\varphi}} = \frac{B\rho^2}{2\Lambda}\,. \tag{2.5}$$

The parameter B is the magnetic field along the z-axis defined by $B^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}|_{\rho=0}$. Since the eleven-dimensional spacetime is flat this metric is expected to be an exact solution of the M-theory including higher derivative terms.[2] However, from the IIA perspective, the Melvin background is always a non-perturbative vacuum because far away from the axis, i.e. when ρ becomes large, the string coupling becomes large and the appropriate description is eleven-dimensional.

For $\rho \ll 1/|B|$ spacetime is approximately flat and the compactification radius is approximately constant. In this region the string coupling is $g = R/\sqrt{\alpha'}$ and perturbative string theory can be trusted provided R is small. In addition, for the Kałuża-Klein ansatz to make sense, the length scale ρ must be much larger than the compactification scale, i.e. $\rho \gg R$. Hence, the condition $|B| \ll 1/R$ is necessary for this background to be a good approximation for IIA theory in a constant R-R magnetic field near $\rho = 0$.

Near the 'critical' magnetic field $|B| \sim 1/R$ the region where space is flat and the theory weakly coupled is much smaller than the compactification radius. This implies that the ten-dimensional interpretation of the solution is not applicable anywhere.

2.1 Spin structuress

Since the angle φ has period 2π the identifications (2.2) are not altered by the replacement $B \to B + n/R$. Hence the magnetic field B describes inequivalent spacetimes within the range [9]

$$-\frac{1}{2R} < B \leq \frac{1}{2R}\,. \tag{2.6}$$

However, the above argument is not quite correct in a theory which includes fermions, like M-theory or superstring theory. In fact we shall see that physics changes drastically under the shift $B \to B + n/R$ for n odd. A fermion acquires a phase equal to -1 under a rotation $\Delta\varphi = 2\pi$ with the identity corresponding to a rotation by 4π. Hence, physics is unaltered only when φ is shifted by 4π. From (2.2) it follows that the spacetime identifications and the boundary conditions on the fermions do not change under the shift $B \to B + 2n/R$. We conclude that inequivalent spacetimes lie within the range

$$-\frac{1}{R} < B \leq \frac{1}{R}\,. \tag{2.7}$$

[2]The Green-Schwarz String propagating in the IIA Melvin background was discussed in [15]

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More formally the Melvin solution has topology $\mathcal{M}^6 \times \mathbb{R}^4 \times S^1$, where $\mathcal{M}^6$ is the topology of the compact six-dimensional space which we shall neglect. In what follows we shall denote the cartesian coordinates in the ρ, ϕ plane of (2.1) by x_1, x_2. The manifold $\mathbb{R}^4 \times S^1$ admits two different spin structures. A vector parallelly transported around the S^1 will be rotated by an angle $\Delta\varphi = 2\pi RB$, while a fermion parallelly transported around the S^1 will return with a phase

$$e^{\pi RB\gamma_{12}} \qquad \text{or} \qquad -e^{\pi RB\gamma_{12}}\,, \tag{2.8}$$

according to the spin structure. Here γ_{12} is a generator of the Spin(3, 1) Lie algebra which generates a rotation in the $x_{1,2}$-plane. We neglect the action of the spin group G due to parallel transport along the compact six-dimensional manifold. Since $(\gamma_{12})^2 = -1$ it is easy to see that for $B \to B + 1/R$ the spin structures are interchanged while for $B \to B + 2/R$ we obtain the same physics.

Although the eleven-dimensional spacetime is flat, the nontrivial identification implies that for $B \neq 0$ no Killing spinors exist, i.e. from the ten-dimensional point of view supersymmetry will be broken by the presence of the magnetic field. Related approaches to supersymmetry breaking include the Scherk-Schwarz mechanism [17, 18] and magnetized tori [19]. Remarkably, from the type-IIA string point of view the argument given above predicts the existence of a maximum magnetic field associated with the R-R 1-form potential

$$|B_{\max}| = \frac{1}{R} = \frac{1}{g\sqrt{\alpha'}}\,. \tag{2.9}$$

This is a non-perturbative prediction and it is a geometrical consequence of the Kałuża-Klein reduction of M-theory to the IIA strings. Also, we expect that under T-duality a similar result holds for the other R-R p-form fields. We think that the existence of a maximum electric/magnetic field ought to have a deep explanation. In fact, for a NS-NS magnetic field on a 3-sphere the same phenomenon occurs [20]. In this case the physical interpretation of the maximum magnetic field is clear, at the critical value the states that couple to the magnetic field become infinitely massive and decouple. It would be interesting to understand better this issue.

2.2 Type-0A strings

The Kałuża-Klein Melvin background admits two different spin structures as described in (2.8). For $B = 0$, the first case in (2.8) corresponds to the supersymmetric IIA compactification of M-theory (fermions obey periodic boundary conditions on S^1), while the second case corresponds to the non-supersymmetric 0A compactification of M-theory (fermions obey anti-periodic boundary conditions on S^1) [16]. As explained in section 2.1, under the shift $B \to B + 1/R$ the spin structures are inverted, therefore it is natural to suspect that the IIA and 0A theories are related

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by such shift of the magnetic field. For example, the IIA theory with critical field B=1/R corresponds to the flat 11D space with the spin structure inverted. Reducing along the Killing vector $l = \partial_{11} + \frac{1}{R}\partial_\varphi$ gives the IIA theory in a critical magnetic field, which is strongly coupled everywhere. Equivalently, reducing along the Killing vector $l' = \partial_{11}$ and adopting the second spin structure in (2.8) gives the 0A theory with vanishing magnetic field.

To be more precise consider the IIA theory with maximum magnetic $B = 1/R$. The radius R is related to the string coupling g by $R = g\sqrt{\alpha'}$. It is important to realize that this is the coupling constant at $\rho = 0$. As explained before, the condition for the coupling to remain small $\rho \ll 1/B = R$ implies that the theory is in the eleven-dimensional regime (although we are keeping g small). From the 0A point of view the situation is rather different: the magnetic field is zero and the string coupling g' satisfies $g' = R/(2\sqrt{\alpha'}) = g/2$ which is small. Hence, the appropriate description of the IIA theory at critical magnetic field, which is strongly coupled, is given by the weakly coupled 0A theory.

More generally, consider the first spin structure in (2.8) and reduce the spacetime described by (2.1) along

$$l = \partial_{11} + B\,\partial_\varphi \tag{2.10}$$

to obtain the IIA theory in a magnetic field B. This is equivalent to adopt the second choice in (2.8) and reduce along

$$l' = \partial_{11} + \left(B \mp \frac{1}{R}\right)\partial_\varphi \tag{2.11}$$

to obtain the 0A theory in a magnetic field $B' = B \mp 1/R$, where the $\mp$ choice correspond to $B > 0$ and $B < 0$, respectively. Hence, we conclude that the IIA theory in a magnetic field B is dual to the 0A theory in a magnetic field $B' = B \mp 1/R$, with the couplings at the z-axis related by $R = g\sqrt{\alpha'} = 2g'\sqrt{\alpha'}$. Notice that the Melvin solution for the 0A theory is similar to the IIA solution (2.5) with the R-R 1-form potential in the untwisted sector. The tachyon and twisted R-R fields are set to zero.

2.3 NS-NS Melvin fluxtube

Because quantization of string theories in R-R backgrounds is not well understood it is difficult to directly prove our conjecture. One can however relate the IIA Melvin background to a background of perturbative string theory by further compactifying the IIA theory with a R-R fluxtube on a circle and by using a '9/11 flip'. The resulting theory will be the nine-dimensional type-II theory with a NS-NS fluxtube.

Applying the construction of section 2 to IIA string theory in ten dimensions relates the flat ten-dimensional spacetime with metric

$$ds^2 = -dt^2 + dy_m dy^m + dz^2 + d\rho^2 + \rho^2 d\varphi^2 + d{x_9}^2\,, \tag{2.12}$$

to a nine-dimensional magnetic fluxtube. Here $m = 1, \ldots, 5$ and x_{11} is replaced by x_9 in the nontrivial identification (2.2). The Kałuża-Klein reduction

$$ds_{10}{}^2 = ds_9{}^2 + e^{2\sigma}\left(dx_9 + 2\mathcal{A}_\mu dx^\mu\right)^2 , \tag{2.13}$$

gives the NS-NS Melvin solution

$$\begin{aligned} ds_9{}^2 &= -dt^2 + dy_m dy^m + dz^2 + d\rho^2 + \Lambda^{-1}\rho^2 d\varphi^2 , \\ A_\varphi &= \frac{B\rho^2}{2\Lambda} , \qquad\qquad e^{2\sigma} \equiv \Lambda = 1 + B^2\rho^2 . \end{aligned} \tag{2.14}$$

The gauge field A_φ is a NS-NS field, coming from the metric element $g_{\mu 9}$ under Kałuża-Klein reduction. Note that the nine-dimensional dilaton field $\phi_9 = \phi - \sigma/2$ is not constant. Since the ten-dimensional dilaton field is trivial and no R-R fields are turned on, it is straightforward to calculate the string spectrum and the partition function of this background. Although the ten-dimensional background is flat the identifications imply: Firstly the string fields in the x_1, x_2-directions are twisted due to the nontrivial rotation in the $x_{1,2}$-plane; Secondly the momentum and winding modes in the x_9-direction have an additional contribution depending on the angular momentum in the $x_{1,2}$-plane.

It can be shown that the quantization of bosonic strings [21] and superstrings [22, 23] moving in the background (2.14) can be reduced to free fields. For the superstring the mass spectrum is given by

$$\begin{aligned} \text{mass}^2 = & \frac{2}{\alpha'}(N_L + N_R) + \frac{m^2}{R^2} + w^2\frac{R^2}{\alpha'^2} - 2B\frac{m}{R}(J_L + J_R) - \\ & -2B\frac{Rw}{\alpha'}(J_L - J_R) + B^2(J_L + J_R)^2 , \end{aligned} \tag{2.15}$$

where m, w are the momentum and winding number of the X_9 circle of radius R and J_L, J_R are the left- and right-moving angular momentum operators in the $x_{1,2}$-plane, respectively. The spectrum is non-supersymmetric for $B \neq 2n/R$ and it contains tachyons coming from the winding sector $w \neq 0$ for $R < \sqrt{2\alpha'}$ and $B \leq B_{\text{crit}} = R/(2\alpha')$. The one loop partition function was also derived in [22, 23] and is of the form

$$\begin{aligned} Z(B,R) = V_7 R \int \frac{d^2\tau}{\tau_2{}^2}\frac{1}{\tau_2{}^3} & \sum_{m,n} \exp\left(-\frac{\pi R^2}{\alpha'\tau_2}\left|m + n\tau\right|^2\right) \\ & \left|\theta\begin{bmatrix} 1 + BRn \\ 1 + BRm \end{bmatrix}(0|\tau)\right|^8 \frac{1}{|\theta_1'(0|\tau)|^6 \left|\theta\left[\begin{smallmatrix} 1+2BRn \\ 1+2BRm \end{smallmatrix}\right](0|\tau)\right|^2} , \end{aligned} \tag{2.16}$$

where $\theta\left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right](v|\tau) = \theta_1(v|\tau)$. In the Green-Schwarz formalism the structure of the partition function becomes very transparent since the twisting of the eight Green-Schwarz fermions and the two bosons in the x_1, x_2-directions are responsible for the twisting of the theta functions in the numerator and denominator of (2.16), respectively.

The properties of theta functions ensure that the partition function $Z(B,R)$ is invariant under $B \to B + 2/R$ and hence the spectrum (2.15) is invariant under this shift. The partition function vanishes when $B = 2n/R$ with $n \in \mathbb{Z}$, indicating that the spectrum is the one of the ten-dimensional IIA string compactified on a circle.

For the critical value of the magnetic field $B = (2n+1)/R$ the spectrum is equivalent to the superstring with anti-periodic boundary conditions for the spacetime fermions [24, 25, 26]. Note that for these values translation invariance is restored in the $x_{1,2}$-plane which introduces a (divergent) volume factor, coming from the twisted bosons. Then the partition function reads

$$\begin{aligned} Z_{\rm crit} = Z(B,R)\mid_{B=\frac{1}{R}} = V_9 R \int \frac{d^2\tau}{{\tau_2}^2}\frac{1}{{\tau_2}^3}\Bigg\{ & Z_{0,0}\frac{|{\theta_3}^4(0|\tau) - {\theta_4}^4(0|\tau) - {\theta_2}^4(0|\tau)|^2}{|\eta(\tau)|^{24}} + \\ & + Z_{1,0}\frac{|\theta_2(0|\tau)|^8}{|\eta(\tau)|^{24}} + Z_{0,1}\frac{|\theta_4(0|\tau)|^8}{|\eta(\tau)|^{24}} + \\ & + Z_{1,1}\frac{|\theta_3(0|\tau)|^8}{|\eta(\tau)|^{24}}\Bigg\}, \end{aligned} \tag{2.17}$$

where Z_{ab} defines the summation over odd and even winding numbers around the two cycles of the torus

$$Z_{ab}(\tau, R) = \sum_{m,n} \exp\left(-\frac{\pi R^2}{\alpha' \tau_2} \mid (2m+a) + (2n+b)\tau \mid^2 \right). \tag{2.18}$$

A Poisson resummation on m of the partition function Z_{ab} defined in (2.18) yields

$$Z_{ab}(\tau, R) = \frac{\sqrt{\alpha' \tau_2}}{2R}\sum_{kl}(-1)^{ak}\exp\left(-\pi\tau_2\left[\frac{\alpha'}{4R^2}k^2 + \frac{R^2}{\alpha'}(2l+b)^2 \right] + \pi i \tau_1 k(2l+b) \right). \tag{2.19}$$

Defining the standard SO(8) characters as follows

$$\chi_o = \frac{1}{\eta(\tau)^{12}}\left({\theta_3}^4 + {\theta_4}^4\right), \qquad \chi_v = \frac{1}{\eta(\tau)^{12}}\left({\theta_3}^4 - {\theta_4}^4\right), \qquad \chi_{s,c} = \frac{1}{\eta(\tau)^{12}}{\theta_2}^4, \tag{2.20}$$

it is straightforward to show that the partition function (2.17) can be expressed in the following way

$$\begin{aligned} Z_{\rm crit} = V_9 R \int \frac{d^2\tau}{{\tau_2}^2}\frac{1}{{\tau_2}^3}\Big\{ & (\chi_v\bar\chi_v + \chi_s\bar\chi_s - \chi_v\bar\chi_s - \chi_s\bar\chi_v)Z_{00} - \\ & - (\chi_v\bar\chi_v + \chi_s\bar\chi_s + \chi_v\bar\chi_s + \chi_s\bar\chi_v)Z_{10} + \\ & + (\chi_o\bar\chi_o + \chi_c\bar\chi_c - \chi_o\bar\chi_c - \chi_c\bar\chi_o)Z_{01} - \\ & - (\chi_o\bar\chi_o + \chi_c\bar\chi_c + \chi_o\bar\chi_c + \chi_c\bar\chi_o)Z_{11}\Big\}. \end{aligned} \tag{2.21}$$

The partition function for the critical Melvin solution (2.21) is the same as for the type-IIA $S_1/(-1)^F\sigma_{1/2}$ orbifold with the identification $R_{\rm orb} = 2R_{\rm Melvin}$. Here F is the spacetime fermion number and $\sigma_{1/2}$ is a shift by half the circumference of the circle.

The first two terms in (2.21) come from the projection onto invariant states and the last two terms are coming from the twisted sector states by modular invariance.

The type-IIA $S_1/(-1)^F\sigma_{1/2}$ orbifold interpolates between type-IIA and 0A theories in the limits $R \to \infty$ and $R \to 0$, respectively. In [16] it was argued that exchanging the orbifold and M-theory circle ('9/11 flip') the ten-dimensional 0A theory is related to a $S_1/(-1)^F\sigma_{1/2}$ orbifold of M-theory. This '9/11 flip' will map the NS-NS fluxtube to a R-R fluxtube, showing that the ten-dimensional 0A theory is related to the IIA theory at the critical R-R magnetic field, as conjectured earlier.

The relation of the type-II NS-NS Melvin solution with magnetic field B and Kałuża-Klein radius R, and the type-IIA $S_1/(-1)^F\sigma_{1/2}$ orbifold with magnetic field $B' = B - 1/R$ (for $B > 0$) and radius $R' = 2R$ can be seen using the lagrangian representation of the Melvin solution (2.16). Then, one can split the sum over m, n into four sectors corresponding to (m, n) being $(e, e), (o, e), (e, o), (o, o)$, these four sectors correspond to the untwisted sector $(1, 1)$ and the three twisted sectors $(g, 1), (1, g), (g, g)$ of the $g = (-1)^F\sigma_{1/2}$ orbifold.

3. Instabilities

Schwinger pair production is mapped under electromagnetic duality to pair production of magnetically charged particles in a constant magnetic field. In Yang-Mills-Higgs theories magnetic monopoles can be pair produced [27, 28], similarly, in a gravitational theory magnetically charged black holes can be pair produced [7]–[10][29]–[32]. For the IIA R-R-Melvin background we expect the production of D6/D$\bar{6}$-brane pairs [10]. Also, we expect some decay mode for the 0A theory. Within the supergravity approximation to M-theory standard semiclassical quantum gravity instanton methods [34] can be applied to estimate the rate for the decay process.[3] All we have to do is to find the instanton associated with the process and calculate its action. Then the rate for the nucleation process is estimated by $\Gamma \sim e^{-I}$. The subsequent lorentzian evolution is determined by a surface of zero extrinsic curvature on the euclidean manifold to which a lorentzian manifold is glued.

3.1 Type-0A and -IIA instantons

The instanton associated with the Kałuża-Klein monopole/anti-monopole pair production is the Myers-Perry Kerr instanton [35] with a single (complexified) angular momentum parameter and six extra flat directions [9]. The metric for this background reads

$$ds^2 = dy_m dy^m + {dx_{11}}^2 + \sin^2\theta\left(r^2 - \alpha^2\right)d\varphi^2 - \frac{\mu}{\rho^2}\left(dx_{11} + \alpha\sin^2\theta\, d\varphi\right)^2 + \\ + \frac{\rho^2}{r^2 - \alpha^2 - \mu}\, dr^2 + \rho^2 d\theta^2 + r^2\cos^2\theta\, d\psi^2\,, \tag{3.1}$$

[3] The evaluation of the one loop determinants in quantum gravity is problematic and will not be discussed here.

where $\rho^2 = r^2 - \alpha^2\cos^2\theta$. There is a coordinate singularity at $r^2 = r_H^2 \equiv \mu + \alpha^2$, the locus of the black hole horizon in the lorentzian metric. The parameters of the background (3.1) can be expressed in terms of

$$B = \frac{\alpha}{\mu}, \qquad \frac{1}{R} = k = \frac{\sqrt{\mu + \alpha^2}}{\mu}, \tag{3.2}$$

where $\omega \equiv i\Omega = iB$ and k are the lorentzian angular velocity and surface gravity, respectively.

We shall reduce the above manifold along the orbits of the vector $l = \partial_{11} + B\,\partial_\varphi$, which has zero norm at $r = r_H$. As in the case of the Melvin solution this can be done by changing to the coordinate $\tilde{\varphi} = \varphi - B\,x_{11}$ and reducing along $l = \partial_{11}$. Then, the conical singularity at $r = r_H$ can be removed provided x_{11} has period $2\pi R$ for fixed $\tilde{\varphi}$. In the limit $r \to \infty$ the instanton metric becomes flat. Because of the identifications on the angles x_{11} and φ the instanton approaches the euclidean Kałuża-Klein Melvin solution describing a decay mode of this background.

From the relations (3.2) it follows that the parameter B lies within, $-1/R \leq B \leq 1/R$. This means that a Kerr instanton with $B > 1/(2R)$ ($B < 1/(2R)$) asymptotically approaches the same magnetic field as the same instanton with parameter $B - 1/R$ ($B + 1/R$). Hence we can define the shifted Kerr instanton with parameters [9]

$$B = \frac{\alpha}{\mu} \mp \frac{1}{R}, \qquad \frac{1}{R} = k = \frac{\sqrt{\mu + \alpha^2}}{\mu}, \tag{3.3}$$

where the $\mp$ signs corresponds to $\alpha > 0$ and $\alpha < 0$, respectively. Both unshifted and shifted instantons (3.2) and (3.3) approach asymptotically the same magnetic field. However, the Melvin backgrounds with magnetic field B and $B \mp 1/R$ have different spin structures. This is reassuring because these two instantons are not identical, allowing the appropriate identification of an instanton as the decay mode of an associated vacuum with given spin structure. In other words, since there are two distinct Kerr instantons approaching asymptotically the same magnetic field the appropriate decay mode is fixed by the spin structure. In fact, both instantons are the same Kerr instanton with different values of its parameters but their ten-dimensional interpretation is quite different.

Since the Kerr instanton has topology $\mathbb{R}^2 \times S^3$, which is connected, it admits a single spin structure. Spinors parallelly transported around an integral curve of l pick a phase $-e^{\pi R\frac{\alpha}{\mu}\gamma}$ for both unshifted and shifted instantons [9]. If we choose the first spin structure for the Melvin background in (2.8), corresponding to the IIA theory, the appropriate Kerr instanton is the shifted one where $\alpha/\mu = B \mp 1/R$. On the other hand, the second choice in (2.8) picks the unshifted instanton as the decay mode for the 0A theory. Remarkably, the spin structures uniquely fix the spacetime decay modes over the physically inequivalent values of the magnetic field for both IIA and 0A theories.

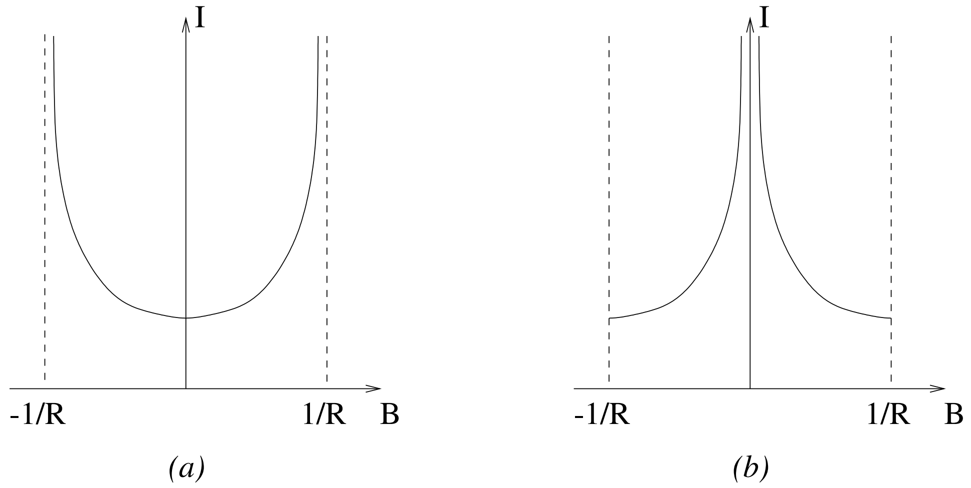

Figure 1: The actions for the instantons associated with the decay of the Melvin background for (a) the 0A theory and (b) the IIA theory. In the former case the decay is through Witten's bubble production while in the latter through D6/$\overline{\text{D6}}$-brane pair production. The actions go to infinity for $B = \pm 1/R$ for the unshifted instanton and for $B = 0$ for the shifted instanton, in agreement with stability of supersymmetric type-IIA strings.

The decay probability rate can be estimated as e^{-I} where the euclidean action for the instantons reads

$$I = -\frac{1}{16\pi G_{11}} \int d^{11}x \sqrt{g}\, R - \frac{1}{8\pi G_{11}} \int d^{10}x \sqrt{h}\, (K - K_0) \ . \tag{3.4}$$

K is the trace of the extrinsic curvature of the boundary with metric h_{ij} and K_0 is the same quantity for the Melvin background. For the unshifted Kerr instanton we have [9]

$$I = \frac{\pi V_6}{8 G_{10}} \frac{R^2}{1 - (BR)^2} \, , \tag{3.5}$$

while for the shifted instanton the action is

$$I = \frac{\pi V_6}{8 G_{10}} \frac{R^2}{1 - (1 - |B|R)^2} \, , \tag{3.6}$$

where V_6 is the volume of the compact six-dimensional manifold. The ten-dimensional Newton constant G_{10} satisfies $16\pi G_{10} = (2\pi)^7 g^2 \alpha'^4$. In figure 1 the action for both instantons as a function of the magnetic field is ploted.

3.2 Decay modes

The euclidean instanton is associated to a tunneling trajectory describing the decay of a spacetime. The post decay evolution depends on a surface of zero extrinsic curvature in the euclidean manifold. This surface provides the initial data to obtain a lorentzian metric describing the spacetime into which the Melvin background decays.

Such a surface is defined by $\psi = 0$ and $\psi = \pi$ with $0 \leq \theta \leq \pi/2$, which is joined at $\theta = \pi/2$. The subsequent lorentzian evolution is obtained by analytic continuation $\psi \to it$. From the eleven-dimensional point of view the interpretation for both instantons is the same. It describes an expanding bubble without spherical symmetry [9], i.e. it is a deformation of Witten's expanding bubble [13] (an expanding cigar). The expanding two-surface $r = r_H$ has initial area $4\pi\mu$ and expands like $\cosh^2 t$. In the case of the 0A theory without magnetic field (or the IIA theory with critical field) the decay mode is indeed given by Witten's expanding bubble corresponding to the euclidean Schwarzchild instanton.

3.2.1 Type 0A

Next we describe the decay process as seen by a ten-dimensional observer. Firstly consider the unshifted instanton associated with the 0A theory. We reduce to ten-dimensions along the vector

$$l = \partial_{11} + B\,\partial_\varphi = \partial_{11} + \Omega\,\partial_\varphi\,, \tag{3.7}$$

where $\Omega = \alpha/\mu$ is the euclidean angular velocity. Because the reduction is co-rotating with the geometry, a single singularity appears at $r = r_H$ where $l^2 = 0$. Of course this is an artifact of the reduction since the geometry is perfectly regular from the eleven-dimensional point of view. The subsequent lorentzian evolution is that of a space falling into the singularity [9].

It is interesting to consider the instanton action for vanishing magnetic field. This will be associated to the bubble nucleation instability of the type-0A theory. The instanton action reads

$$I = \frac{\pi V_6}{8G_{10}} R^2 = 4\pi \frac{V_6}{(2\pi)^6 \alpha'^3}\,, \tag{3.8}$$

which does not depend on the ten-dimensional string coupling g'. In the case that $V_6 \ll \alpha'^3$, i.e. when the theory is effectively four-dimensional, one might think that the decay of spacetime through bubble production is dominant. In this case the length scale R is of order of the Planck length and the semi-classical approximation is no longer valid. However, we still expect this to indicate an instability. In fact, we can consider the T-dual theory by dualizing the 6-torus. Then the false vacuum is long lived and the semi-classical calculation reliable.

3.2.2 Type IIA

Secondly consider the shifted instanton associated with the IIA theory. Now we reduce along the orbits of the vector

$$l = \partial_{11} + B\,\partial_\varphi = \partial_{11} + \left(\Omega \mp \frac{1}{R}\right)\partial_\varphi\,. \tag{3.9}$$

The horizon $r = r_H$ becomes a line horizon where the metric is regular except for the two singularities at the ends ($\theta = 0, \pi$) [9]. This line horizon becomes a cigar from the 11D perspective. Subsequently the singularities will accelerate apart from each other (corresponding to the ends of an expanding cigar in 11D).

If $|B| \ll 1/R$ the Melvin solution describes the perturbative IIA theory in a constant magnetic R-R field within a region $\rho \ll R \ll 1/|B|$. Hence it is natural to ask whether the creation of D6/$\bar{\text{D}}$6-brane pairs as described by semiclassical gravity follows the expected behavior for the Schwinger pair production. In the limit $|B| \ll 1/R$ the action (3.6) becomes

$$I = \frac{\pi V_6}{8G_{10}} \frac{R}{2|B|} = \frac{\pi M_6}{2|B|}, \tag{3.10}$$

where $M_6 = V_6 T_6$ and $T_6 = ((2\pi)^6 \alpha'^{7/2} g)^{-1}$ is the D6-brane tension. This gives exactly the Schwinger nucleation rate $\Gamma \sim e^{-\pi m^2/qE}$ for a particle with charge $q = 2m$ in a constant electric field E. In fact, the D6-brane action in the perturbative IIA theory region is for vanishing worldvolume gauge field (notice that in our conventions $\rho_6 = 2T_6$)

$$S = -T_6 \int d^7\sigma\, e^{-\phi} \sqrt{-\det \hat{G}_{\alpha\beta}} - 2T_6 \int \hat{\mathcal{A}}_7 , \tag{3.11}$$

where $\hat{G}_{\alpha\beta}$ and $\hat{\mathcal{A}}_7$ are the pull-backs to the worldvolume of the metric and dual R-R 7-form potential ($\star\mathcal{F}_2 = d\mathcal{A}_7$), respectively. A short calculation shows that in the Melvin background

$$d\mathcal{A}_7 = B\; dt \wedge dz \wedge dy_1 \wedge \cdots \wedge dy_6 \,. \tag{3.12}$$

This indicates that the instanton calculation gives exactly the expected result for a BPS saturated D6-brane subjected to a constant electric field of magnitude $E = B$.

The magnetically charged objects which will be created are D6-$\bar{\text{D}}$6 pairs wrapped on the T^6. Hence, from the four-dimensional point of view these are D0-branes which couple electrically to the dual of the magnetic field. Then the action of the instanton can be easily reproduced in a simple probe calculation of the D0 brane in the Melvin background. Assume that the D-particle moves only in the (euclidean) two plane spanned by τ and z and that the only non-vanishing component of the electromagnetic tensor is $F_{\tau z} = B$. The Born Infeld action for the euclidean signature reads

$$S = M_6 \int d\sigma \left(\sqrt{\dot{\tau}^2 + \dot{z}^2} - Bz\dot{\tau} \right). \tag{3.13}$$

The equations of motion following from this action are

$$\frac{\partial}{\partial\sigma}\left(\frac{\dot{\tau}}{\sqrt{\dot{\tau}^2 + \dot{z}^2}} - Bz \right) = 0\,, \qquad \frac{\partial}{\partial\sigma}\left(\frac{\dot{z}}{\sqrt{\dot{\tau}^2 + \dot{z}^2}} + B\tau \right) = 0\,. \tag{3.14}$$

The integration constant can be set to zero by shifting τ, z and a solution of these equations is given by

$$\tau = \frac{1}{B}\sin\sigma\,, \qquad z = \frac{1}{B}\cos\sigma\,. \tag{3.15}$$

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This solution parameterizes a circle of radius $1/B$ in the range $\sigma \in [0, 2\pi]$. Evaluating the action gives

$$S = \frac{M_6}{|B|} \int_0^{2\pi} d\sigma \left(1 - \cos^2 \sigma\right) = \frac{\pi M_6}{2|B|}\,, \tag{3.16}$$

which agrees exactly with (3.10).

3.3 Non-compact spacetime

So far our discussion considered a ten-dimensional spacetime where six dimensions live on a T^6, i.e. we focused on the compactification to four dimensions. However, the results above reported can be generalized to theories with a non-compact ten-dimensional spacetime. We shall describe briefly such generalization and refer the reader to [10] for the details.

In the case of a non-compact type-IIA Melvin universe described by (2.5) the instanton action (3.6) for the D6/D$\bar{6}$ pair production is infinite because the volume V_6 is infinite. However, the Melvin vacuum can decay via the nucleation of a single spherical D6-brane. The appropriate instanton is the Myers-Perry eleven-dimensional shifted instanton with a single (complexified) angular momentum. For small magnetic field $|B| \ll 1/R$, the action agrees with that of a spherical probe with action given by (3.11) calculated in the perturbative IIA region. At critical magnetic field $|B| = 1/R$, the ten-dimensional type-IIA description breaks down and the appropriate description is given by weakly coupled type-0A theory. In this case we consider the eleven-dimensional unshifted Schwarzschild instanton that describes the bubble (a 8-sphere) nucleation in the type-0A vacuum. The action is

$$I = \frac{2^{10}\Omega_7}{G_{10}} R^8 = \frac{2^{15}\Omega_7}{\pi^6} g'^6\,, \tag{3.17}$$

where Ω_7 is the unit 7-sphere volume. As for the compact spacetime there are two important regimes. When the compactification radius is large compared to the Planck length the false vacuum is long lived and spacetime will eventually decay through bubble production. This happens at the strong coupling regime where the 0A tachyon is expected to become massive [16]. When the compactification radius is of the order of the Planck length the semi-classical approximation breaks down. At this point one expects the perturbative 0A tachyon to appear in the spectrum signaling the instability of the theory. The relation of nonperturbative instabilities and the appearance of tachyons in the perturbative spectrum has been discussed in several other contexts [36, 37, 38].

4. Conclusion

In this paper we discussed some aspects of the Kałuża-Klein Melvin Solution in M-theory. Remarkably this quite nontrivial ten dimensional fluxtube background can be reproduced by compactifying the flat eleven dimensional space, provided the identification of points used for the dimensional reduction includes a rotation on a two-plane.

We have uncovered a relation between the Melvin background and type-IIA and -0A strings. This connection arose as a consequence of the M-theory Kałuża-Klein reduction to IIA and 0A strings. An immediate consequence of the dimensional reduction is the existence of a maximum magnetic field. Then a careful analysis of the spacetime spin structures led us to conjecture that under a shift of the Melvin magnetic field both IIA and 0A theories are equivalent. Evidence for this relation was given by a further compactification on a circle and by using perturbative string theory and the '9/11' flip symmetry of M-theory. We proceeded by analyzing the instabilities of both IIA and 0A Melvin solutions. In each case and for physically inequivalent magnetic fields there is an instanton associated to the decay of spacetime. In the case of the IIA theory this instanton is associated to the nucleation of D6/D$\bar{6}$-brane pairs, while in the 0A case to Witten's expanding bubble. At weak coupling type-OA theories also have a tachyon, which signals a perturbative instability, it is an interesting question what the interpretation of this tachyon on the IIA side is, in the following we offer some speculations in this direction.

Consider the nucleation of a brane/anti-brane pair in the type-IIA theory. The typical distance δ between the nucleated brane and anti-brane can be estimated by associating the area $A = 4\pi\mu$ of the cigar to the initial mass of a membrane connecting the monopole/anti-monopole pair. The membrane mass is given by $M = A\,T_m$, where T_m is the membrane tension. Then the distance δ can be estimated by setting $M = \delta\,T_s$, where the string tension is related to the membrane tension by $T_s = 2\pi R\,T_m$. For small values of the magnetic field $|B| \ll 1/R$, we have $A = \pi R/|B|$ and the typical distance between the D6/D$\bar{6}$ pair is $\delta \sim 1/(2|B|) \gg R$. On the other hand, for the critical value of the magnetic field $|B| = 1/R$ the area of the cigar is given by $A = 4\pi R^2$. Hence the classical mass for the strings ending on the D6/D$\bar{6}$-branes is $M = 2g\sqrt{\alpha'}\,T_s$. This mass vanishes in perturbative string theory. Quantizing the open strings stretching between the D6/D$\bar{6}$ pair one obtains the usual string tachyon. However, for larger values of g this tachyon acquires a positive classical mass which is associated with a minimal distance between the D6/D$\bar{6}$ pair given by $\delta \sim 2g\sqrt{\alpha'}$. As explained before, for the IIA theory at critical magnetic field the appropriate weakly coupled dual description is given by the type-0A theory with vanishing magnetic field and string coupling $g' = g/2$. In this case spacetime decays through bubble nucleation and the string tachyon becomes massive at strong coupling.

Finally we comment on the analysis by Sen [33] of the four-dimensional Kerr instanton with one flat time direction and six flat spatial directions. After reduction this solution is interpreted as a D6/D$\bar{6}$-brane pair kept in precarious equilibrium in a R-R magnetic field. Sen interpreted the case of a critical magnetic field $|B| = 1/R$

as a coincident brane/anti-brane pair because reducing along a different direction the critical field can be brought to a vanishing magnetic field. In the light of the discussion above, the appropriate weakly coupled description is given by the type-0A theory. The classical mass of the string tachyon ending on the D6/$\bar{\text{D6}}$ pair is now interpreted as the mass of the 0A tachyon, in the same spirit of the previous paragraph.

Acknowledgments

We wish to thank C. Bachas, L. Cornalba and C. Kounnas for helpful comments. The work of M.G. was supported in part by the David and Lucile Packard Foundation. The work of M.S.C was supported by a Marie Curie Fellowship under the European Commission's Improving Human Potential programme.

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References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [`hep-th/9711200`].

[2] N. Seiberg, L. Susskind and N. Toumbas, *Strings in background electric field, space/time noncommutativity and a new noncritical string theory*, *J. High Energy Phys.* **06** (2000) 021 [`hep-th/0005040`].

[3] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, *S-duality and noncommutative gauge theory*, *J. High Energy Phys.* **06** (2000) 036 [`hep-th/0005048`].

[4] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *OM theory in diverse dimensions*, *J. High Energy Phys.* **08** (2000) 008 [`hep-th/0006062`].

[5] E. Bergshoeff, D.S. Berman, J.P. van der Schaar and P. Sundell, *Critical fields on the M5-brane and noncommutative open strings*, *Phys. Lett.* **B 492** (2000) 193 [`hep-th/0006112`].

[6] M.A. Melvin, *Pure magnetic and electric geons*, *Phys. Lett.* **8** (1964) 65.

[7] F. Dowker, J.P. Gauntlett, D.A. Kastor and J. Traschen, *Pair creation of dilaton black holes*, *Phys. Rev.* **D 49** (1994) 2909 [`hep-th/9309075`].

[8] F. Dowker, J.P. Gauntlett, S.B. Giddings and G.T. Horowitz, *On pair creation of extremal black holes and Kałuża-Klein monopoles*, *Phys. Rev.* **D 50** (1994) 2662 [`hep-th/9312172`].

[9] F. Dowker, J.P. Gauntlett, G.W. Gibbons and G.T. Horowitz, *The decay of magnetic fields in Kałuża-Klein theory*, *Phys. Rev.* **D 52** (1995) 6929 [`hep-th/9507143`].

[10] F. Dowker, J.P. Gauntlett, G.W. Gibbons and G.T. Horowitz, *Nucleation of p-branes and fundamental strings*, *Phys. Rev.* **D 53** (1996) 7115 [`hep-th/9512154`].

[11] L.J. Dixon and J.A. Harvey, *String theories in ten-dimensions without space-time supersymmetry*, *Nucl. Phys.* **B 274** (1986) 93.

[12] N. Seiberg and E. Witten, *Spin structures in string theory*, *Nucl. Phys.* **B 276** (1986) 272.

[13] E. Witten, *Instability of the Kałuża-Klein vacuum*, *Nucl. Phys.* **B 195** (1982) 481.

[14] G.W. Gibbons and K. Ichi Maeda, *Black holes and membranes in higher dimensional theories with dilaton fields*, *Nucl. Phys.* **B 298** (1988) 741.

[15] J.G. Russo and A.A. Tseytlin, *Green-schwarz superstring action in a curved magnetic Ramond-Ramond background*, *J. High Energy Phys.* **04** (1998) 014 [hep-th/9804076].

[16] O. Bergman and M.R. Gaberdiel, *Dualities of type-0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[17] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60.

[18] C. Kounnas and B. Rostand, *Coordinate dependent compactifications and discrete symmetries*, *Nucl. Phys.* **B 341** (1990) 641.

[19] C. Bachas, *A way to break supersymmetry*, hep-th/9503030.

[20] E. Kiritsis and C. Kounnas, *Infrared behavior of closed superstrings in strong magnetic and gravitational fields*, *Nucl. Phys.* **B 456** (1995) 699 [hep-th/9508078].

[21] J.G. Russo and A.A. Tseytlin, *Constant magnetic field in closed string theory: an exactly solvable model*, *Nucl. Phys.* **B 448** (1995) 293 [hep-th/9411099].

[22] A.A. Tseytlin, *Closed superstrings in magnetic flux tube background*, *Nucl. Phys.* **49** *(Proc. Suppl.)* (1996) 338 [hep-th/9510041].

[23] J.G. Russo and A.A. Tseytlin, *Magnetic flux tube models in superstring theory*, *Nucl. Phys.* **B 461** (1996) 131 [hep-th/9508068].

[24] R. Rohm, *Spontaneous supersymmetry breaking in supersymmetric string theories*, *Nucl. Phys.* **B 237** (1984) 553.

[25] J.J. Atick and E. Witten, *The hagedorn transition and the number of degrees of freedom of string theory*, *Nucl. Phys.* **B 310** (1988) 291.

[26] M.B. Green, *Wilson-Polyakov loops for critical strings and superstrings at finite temperature*, *Nucl. Phys.* **B 381** (1992) 201.

[27] I.K. Affleck and N.S. Manton, *Monopole pair production in a magnetic field*, *Nucl. Phys.* **B 194** (1982) 38.

[28] I.K. Affleck, O. Alvarez and N.S. Manton, *Pair production at strong coupling in weak external fields*, *Nucl. Phys.* **B 197** (1982) 509.

[29] G.W. Gibbons, *Quantized flux tubes in Einstein-Maxwell theory and noncompact internal spaces*, Print-86-0411 (Cambridge) Presented at 22nd Karpacz Winter School of Theoretical Physics *Fields and geometry*, Karpacz, Poland, Feb 17 - Mar 1, 1986.

[30] D. Garfinkle and A. Strominger, *Semiclassical Wheeler wormhole production*, *Phys. Lett.* **B 256** (1991) 146.

[31] D. Garfinkle, S.B. Giddings and A. Strominger, *Entropy in black hole pair production*, *Phys. Rev.* **D 49** (1994) 958 [gr-qc/9306023].

[32] S.W. Hawking and S.F. Ross, *Duality between electric and magnetic black holes*, *Phys. Rev.* **D 52** (1995) 5865 [hep-th/9504019].

[33] A. Sen, *Strong coupling dynamics of branes from M-theory*, *J. High Energy Phys.* **10** (1997) 002 [hep-th/9708002].

[34] S.W. Hawking and G.W. Gibbons, *Euclidean quantum gravity*, World Scientific 1993.

[35] R.C. Myers and M.J. Perry, *Black holes in higher dimensional space-times*, *Ann. Phys. (NY)* **172** (1986) 304.

[36] C.G. Callan Jr. and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[37] M. Fabinger and P. Hořava, *Casimir effect between world-branes in heterotic M-theory*, *Nucl. Phys.* **B 580** (2000) 243 [hep-th/0002073].

[38] G.T. Horowitz and L. Susskind, *Bosonic M-theory*, hep-th/0012037.

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